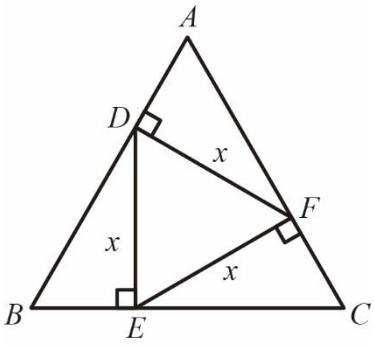
(63)多邊形和三角

(1)



ΔABC和ΔDEF都是正三角形，$\overline{AB}=\overline{BC}=\overline{AC}=1$，$\overline{DF}\perp\overline{AD}$，$\overline{DE}\perp\overline{BC}$，$\overline{EF}\perp\overline{AC}$，求$x$

$\angle DBE=60°$，$\therefore\overline{DB}=\frac{x}{\sin 60°}=\frac{x}{\frac{\sqrt{3}}{2}}=\frac{2}{\sqrt{3}}x$

$\angle DAF=60°$，$\therefore\overline{AD}=\frac{x}{\cos 60°}=\frac{x}{\frac{1}{2}}=2x$

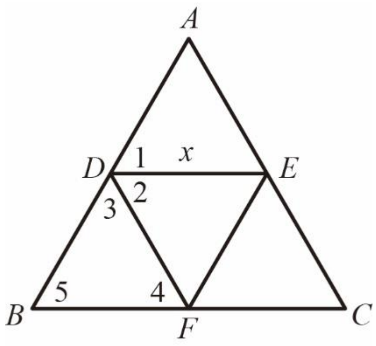
$\because\overline{AD}+\overline{DB}=1$

$\therefore 2x+\frac{2}{\sqrt{3}}x=1$

$(2+\frac{2}{\sqrt{3}})x=1$

$$x=\frac{1}{2+\frac{2}{\sqrt{3}}}=\frac{\sqrt{3}}{2\sqrt{3}+2}=\frac{\sqrt{3}(2\sqrt{3}-2)}{(2\sqrt{3}+2)(2\sqrt{3}-2)}=\frac{6-2\sqrt{3}}{12-4}=\frac{2(3-\sqrt{3})}{8}=\frac{3-\sqrt{3}}{4}$$

(2)



三角形ABC是一正三角形，每邊長度為1，ΔDFE也是一正三角形，$\overline{DE} \parallel \overline{BC}$，求$x$

因為ΔABC是正三角形，$\angle ABC = 60°$

因為$\overline{DE} \parallel \overline{BC}$，$\angle ADE = 60°$，因此$\angle AED = 60°$

$\therefore \angle DAE = \angle ADE = \angle AED = 60°$

$\therefore \Delta ADE$為一正三角形

$\overline{DE} = x = \overline{AD}$

$\angle 1 + \angle 3 = 120°$

$\angle 3 + \angle 4 = 120°$

$\therefore \angle 1 = \angle 4 = 60°$

因為ΔABC為正三角形，故$\angle 5 = 60°$

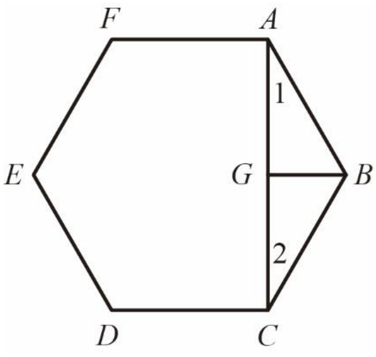
$\therefore \angle 3 = 180° - \angle 4 - \angle 5 = 180° - 60° - 60° = 60°$

$\therefore \Delta DBF$為一正三角形，$\overline{DB} = \overline{DF} = x$

$\overline{DB} + \overline{DA} = x + x = 2x = 1$

$\therefore x = \dfrac{1}{2}$

(3)



$ABCDEF$是一正六邊形，每邊長為1，求$\overline{AC}$

因為$\overline{AB}=\overline{BC}$，故$\Delta ABC$為一等腰三角形

$\therefore \angle 1=\angle 2$

如果正多邊形的邊數為n，則內角$=\frac{(n-2)180°}{n}$

$\therefore \angle ABC=\frac{(6-2)180°}{6}=120°$

$\therefore \angle 1=\angle 2=\frac{1}{2}(180°-120°)=\frac{60°}{2}=30°$

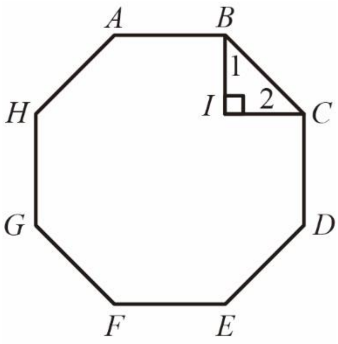
作$\overline{BG}\perp\overline{AC}$

可證$\Delta ABG\cong\Delta CBG$且$\overline{AG}=\overline{CG}$

$\therefore \overline{AG}=\overline{AB}\cos 30°=\cos 30°=\frac{\sqrt{3}}{2}$

$\overline{AC}=2\overline{AG}=\sqrt{3}$

(4)



上圖是一個正八邊形，每邊長為1，$\overline{BI} \perp \overline{AB}$，$\overline{IC} \perp \overline{BI}$，求$\overline{BI}$

正八邊形的內角$= \frac{(8-2)180°}{8} = 135°$

$\angle ABI = 90°$，$\therefore \angle 1 = 45°$

因為$\angle BIC = 90°$，$\therefore \angle 2 = 180° - 90° - 45° = 45°$

$\therefore \overline{BI} = \overline{CI}$

令$\overline{BI} = x$

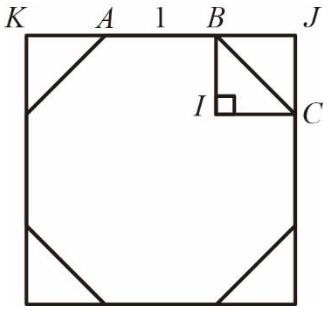
$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \frac{1}{\sqrt{2}}$

$\therefore \overline{BI} = \frac{1}{\sqrt{2}}$

(5)



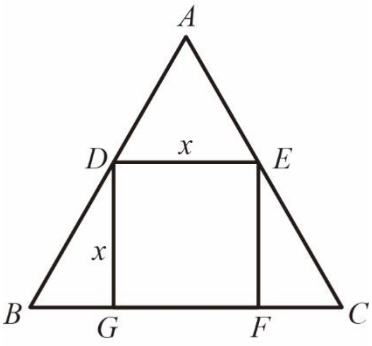
承上題，求正八邊形的外接正方形的邊長

如上圖，$\overline{BI} = \overline{CI} = \frac{1}{\sqrt{2}}$

$\therefore \overline{JK} = \overline{AB} + 2\overline{BJ} = 1 + \frac{2}{\sqrt{2}}$

$\therefore$外接正方形的邊長是$1 + \frac{2}{\sqrt{2}}$

（6）



ΔABC為一正三角形，$DEFG$為一正方形，$\overline{DE} \parallel \overline{BC}$，求$x$

$\because \overline{DE} \parallel \overline{BC}$，$\therefore \angle ADE = 60°$

$\because \Delta ABC$為一正三角形，$\therefore \Delta ADE$為一正三角形

$\therefore \overline{AD} = x$

ΔDBG中，$\angle B = 60°$，$\overline{DG} = x$

$$\therefore \overline{DB} = \frac{x}{\sin 60°} = \frac{x}{\frac{\sqrt{3}}{2}} = \frac{2x}{\sqrt{3}}$$

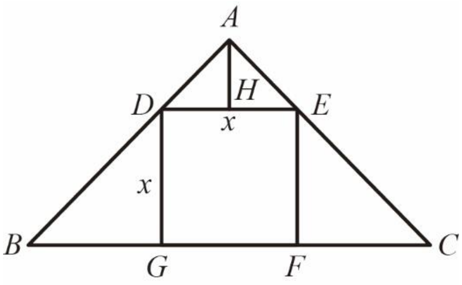
$$\overline{AD} + \overline{DB} = x + \frac{2x}{\sqrt{3}} = 1$$

$$x\left(1 + \frac{2}{\sqrt{3}}\right) = 1$$

$$x\left(\frac{\sqrt{3}+2}{\sqrt{3}}\right) = 1$$

$$\therefore x = \frac{\sqrt{3}}{\sqrt{3}+2} = \frac{\sqrt{3}(\sqrt{3}-2)}{(\sqrt{3}+2)(\sqrt{3}-2)} = \frac{3-2\sqrt{3}}{3-4} = 2\sqrt{3} - 3$$

(7)



ΔABC為一等腰三角形，$\angle B = \angle C = 45°$，$\overline{AB} = \overline{AC} = 1$，$DEFG$為一正方形，$\overline{DE} \parallel \overline{BC}$，求$x$

ΔDBG中，$\angle B = 45°$

$\therefore \overline{BG} = \overline{DG}$

$\therefore \overline{DB}^2 = 2x^2$

$\overline{DB} = \sqrt{2}x$

作$\overline{AH} \perp \overline{DE}$，可證$\overline{DH} = \frac{x}{2}$

$\therefore \overline{AD}^2 = 2(\frac{x}{2})^2 = \frac{x^2}{2}$

$\overline{AD} = \frac{x}{\sqrt{2}}$

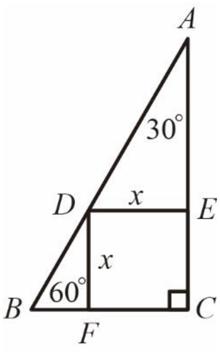
$\overline{AD} + \overline{DB} = \frac{x}{\sqrt{2}} + \sqrt{2}x = 1$

$x(\frac{1}{\sqrt{2}} + \sqrt{2}) = 1$

$x\left(\frac{1+2}{\sqrt{2}}\right) = 1$

$x = \frac{\sqrt{2}}{3}$

(8)



ΔABC為一直角三角形，$\angle B = 60°$，$\angle C = 90°$，$\overline{DE} \parallel \overline{BC}$，$\overline{AB} = 1$，$DECF$為一正方形，求$x$

ΔDBF中，$\overline{DF} = x$，$\therefore DB = \frac{x}{\sin 60°} = \frac{2x}{\sqrt{3}}$

ΔADE中，$\overline{AD} = \frac{x}{\sin 30°} = \frac{x}{\frac{1}{2}} = 2x$

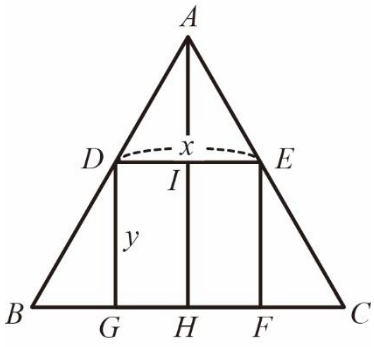
$$AD + DB = 2x + \frac{2}{\sqrt{3}}x = 1$$

$$2x\left(1 + \frac{1}{\sqrt{3}}\right) = 1$$

$$2x\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) = 1$$

$$x = \frac{\sqrt{3}}{2(\sqrt{3}+1)} = \frac{\sqrt{3}(\sqrt{3}-1)}{2(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3-\sqrt{3}}{2(3-1)} = \frac{3-\sqrt{3}}{4}$$

(9)



ΔABC為一正三角形，每邊長度為1，$DEFG$為一長方形，$\overline{DE} \parallel \overline{BC}$，已知$\overline{DE} = x$，求$\overline{DG}$

作$\overline{AH} \perp \overline{BC}$，$\overline{AH}$與$\overline{BC}$交於$I$

可以證明$\Delta AID \cong \Delta AIE$，$\therefore \overline{DI} = \frac{x}{2}$

$\angle DAI = 30°$

$$\frac{\overline{DI}}{\overline{AI}} = \tan 30° = \frac{1}{\sqrt{3}}$$

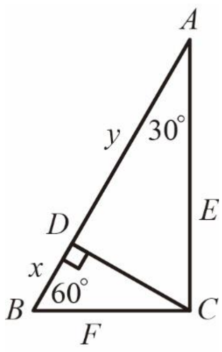
$$\therefore \overline{DI} = \frac{1}{\sqrt{3}}\overline{AI} = \frac{2x}{\sqrt{3}}$$

$\angle ABH = 60°$，$\overline{AH} = \overline{AB}\sin 60° = \frac{\sqrt{3}}{2}$

$$\therefore \overline{IH} = \overline{AH} - \overline{AI} = \frac{\sqrt{3}}{2} - \frac{2x}{\sqrt{3}}$$

$$\therefore y = \frac{3-4x}{2\sqrt{3}}$$

（10）



ΔABC中，$\angle A = 30°$，$\angle B = 60°$，$\angle C = 90°$，$\overline{AB} = 1$，$\overline{CD} \perp \overline{AB}$，$\overline{DB} = x$，$\overline{DA} = y$，求$x$和$y$

ΔDBC中，$\frac{\overline{DC}}{x} = \tan 60° = \sqrt{3}$

$\therefore x = \frac{\overline{DC}}{\sqrt{3}}$

ΔADC中，$\frac{\overline{DC}}{y} = \tan 30° = \frac{1}{\sqrt{3}}$，$\therefore y = \sqrt{3}\overline{DC}$

$$\frac{x}{y} = \frac{\frac{\overline{DC}}{\sqrt{3}}}{\sqrt{3}\overline{DC}} = \frac{\overline{DC}}{\sqrt{3}\sqrt{3}\overline{DC}} = \frac{1}{3}$$

$\therefore y = 3x$

$x + y = 1$

$x + 3x = 1$

$x = \frac{1}{4}$

$y = \frac{3}{4}$