

(26) 指數

假設 a 為實數， n 是正整數

$a^n = \overbrace{a \cdot a \cdots a}^{n \text{ 次}}$ ， a 是底數， n 是指數

$$(1) 2^3 = 2 \times 2 \times 2 = 4 \times 2 = 8$$

$$(2) 3^4 = 3 \times 3 \times 3 \times 3 = 27 \times 3 \times 3 = 81 \times 3 = 243$$

$$(3) 4^3 = 4 \times 4 \times 4 = 16 \times 4 = 64$$

$$(4) \left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$(5) \left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{9} \times \frac{2}{3} = \frac{8}{27}$$

$$(6) (-1)^2 = (-1) \times (-1) = 1$$

$$(7) (-1)^3 = (-1) \times (-1) \times (-1) = -1$$

$$(8) \left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = \frac{1}{4} \times \left(-\frac{1}{2}\right) = -\frac{1}{8}$$

指數定律

指數第一定律 $a^m a^n = a^{m+n}$

$$(9) a = 2, m = 2, n = 3$$

$$(2^2)(2^3) = (2 \times 2) \times (2 \times 2 \times 2) = 4 \times 8 = 32$$

我們可以用 $(a^m)(a^n) = a^{m+n}$ 來計算

$$(a^2)(a^3) = a^{2+3} = a^5 = 32$$

$$(10) a = \frac{1}{2}, m = 2, n = 2$$

$$\left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{2+2} = \left(\frac{1}{2}\right)^4 = \frac{1^4}{2^4} = \frac{1}{16}$$

$$(11) a = -2, m = 1, n = 2$$

$$(-2)^1 (-2)^2 = (-2)^{1+2} = (-2)^3 = -8$$

$$a^m a^n = a^{m+n} \text{ 也可以寫成 } a^{m+n} = (a^m)(a^n)$$

假設我們要計算 a^h ，而且很大，就可以利用以上的定律

$$(12) \text{ 假設 } a = 2, h = 7, \text{ 我們可以將 } h \text{ 分解成 } h = 7 = 3 + 4$$

$$\therefore a^h = 2^7 = (2^3)(2^4)$$

$$\text{要求 } 2^3, \text{ 我們又可以使用 } 3=2+1 \therefore 2^3 = (2^2)(2^1)$$

$$\text{要求 } 2^4, \text{ 我們可以利用 } 4=2+2 \therefore 2^4 = (2^2)(2^2)$$

$$\therefore 2^7 = (2^3)(2^4) = (2^2)(2^1)(2^2)(2^2) = 4 \times 2 \times 4 \times 4 = 8 \times 16 = 128$$

(13) 假設 $a = 3$, $h = 5$, $h = 2 + 3$

$$\therefore 3^h = 3^5 = (3^2)(3^3)$$

$$3^3 = (3^1)(3^2)$$

$$\therefore 3^h = 3^5 = (3^2)(3^1)(3^2) = 9 \times 3 \times 9 = 27 \times 9 = 243$$

(14) 求 2^{10}

$$2^{10} = (2^5)(2^5) = (2^3)(2^2)(2^3)(2^2) = 8 \times 4 \times 8 \times 4 = 32 \times 32 = 1024$$

指數第二定律 $\frac{a^m}{a^n} = a^{m-n}$

假設 m, n 都是正整數，而且 $m > n$ ，則我們可以假設 $m = n + h$ ，因此 $m - n = h$ 。

根據第一定律 $a^m = a^{n+h} = a^n a^h$

$$\therefore \frac{a^m}{a^n} = \frac{a^n a^h}{a^n} = a^h = a^{m-n}$$

(15) $a=2$, $m=5$, $n=3$

$$\frac{a^m}{a^n} = \frac{a^5}{a^3} = a^{5-3} = a^2 = 2^2 = 4$$

(16) $a = -3$, $m = 4$, $n = 2$

$$\frac{a^m}{a^n} = a^{m-n} = (-3)^{4-2} = (-3)^2 = 9$$

(17) $a = \frac{3}{2}$, $m = 5$, $n = 4$

$$\frac{a^m}{a^n} = a^{m-n} = a^{5-4} = a^1 = \left(\frac{3}{2}\right)^1 = \frac{3}{2}$$

$$(18) a = 3, m = 7, n = 5$$

$$\frac{a^m}{a^n} = a^{m-n} = a^{7-5} = a^2 = 3^2 = 9$$

指數第三定律 $a^0 = 1$

從指數第二定律，我們有 $a^{m-n} = \frac{a^m}{a^n}$

假設 $m=n$ ， $a^{m-n} = a^0 = \frac{a^m}{a^m} = 1$

$$\therefore a^0 = 1$$

$$(19) (\pi^5 + \pi^3)^0 = 1$$

指數第四定律 $a^{-n} = \frac{1}{a^n}$

我們先看幾個例子

$$(20) a = 2, m = 2, n = 3, n - m = 3 - 2 = 1$$

$$a^{m-n} = \frac{a^m}{a^n} = \frac{a^2}{a^3} = \frac{a^2}{(a^2)(a^1)} = \frac{1}{a^1} = \frac{1}{2} = \frac{1}{2^{n-m}}$$

$$(21) a = 3, m = 2, n = 4, n - m = 2$$

$$a^{m-n} = \frac{a^m}{a^n} = \frac{a^2}{a^4} = \frac{a^2}{(a^2)(a^2)} = \frac{1}{a^2} = \frac{1}{3^2} = \frac{1}{3^{n-m}}$$

假設 $n > m$ ，可令 $n = m + h$

$$a^{m-n} = \frac{a^m}{a^n} = \frac{a^m}{(a^m)(a^h)} = \frac{1}{a^h}$$

但 $m - n = -h$

$$\therefore a^{-h} = \frac{1}{a^h}$$

以上是指數第四定律的證明，我們也可以用指數第一定律和指數第三定律證明。

根據指數第一定律 $(a^n)(a^{-n}) = a^{n-n} = a^0$

根據指數第三定律 $a^0 = 1$

$$\therefore (a^n)(a^{-n}) = 1$$

$$\therefore (a^{-n}) = \frac{1}{a^n}$$

$$(22) 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$(23) 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$(24) 5^{-3} = \frac{1}{5^3} = \frac{1}{125}$$

指數第五定律 $(a^n)(b^n) = (ab)^n$

假設 $a = 2$, $b = 3$, $n = 2$

$$(a^n)(b^n) = (2^2)(3^2) = 4 \times 9 = 36$$

如果利用指數第五定律

$$(2^2)(3^2) = (2 \times 3)^2 = 6^2 = 36$$

$$(25) (3^3)(2^3) = (3 \times 2)^3 = 6^3 = 216$$

$$(26) (-3)^2(2)^2 = ((-3) \times 2)^2 = (-6)^2 = 36$$

$$(27) (-2)^3(2)^3 = ((-2) \times 2)^3 = (-4)^3 = -64$$

$$(28) \left(\frac{2}{3}\right)^2\left(\frac{1}{2}\right)^2 = \left(\frac{2}{3} \times \frac{1}{2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$