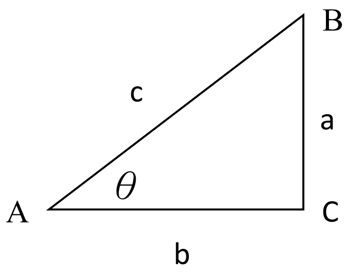
(21) 三角函數的公式一覽表

基本公式

銳角



1. $sin\theta = \frac{a}{c} = \frac{a}{\sqrt{a^2+b^2}}$

2. $cos\,\theta = \frac{b}{c} = \frac{b}{\sqrt{a^2+b^2}}$

3. $tan\,\theta = \frac{a}{b}$

4. $csc\,\theta = \frac{1}{sin\,\theta} = \frac{\sqrt{a^2+b^2}}{a}$

5. $sec\,\theta = \frac{1}{cos\,\theta} = \frac{\sqrt{a^2+b^2}}{b}$

6. $cot\,\theta = \frac{1}{tan\,\theta} = \frac{b}{a}$

7. $a = c\,sin\,\theta$
8. $b = c\,cos\,\theta$

廣義角

9. $sin(2n\pi \pm \theta) = \pm\,sin\,\theta$
10. $cos(2n\pi \pm \theta) = \pm\,cos\,\theta$
11. $tan(n\pi \pm \theta) = \pm\,tan\,\theta$
12. $cot(n\pi \pm \theta) = \pm\,cot\,\theta$
13. $sec(2n\pi \pm \theta) = \pm\,sec\,\theta$
14. $csc(2n\pi \pm \theta) = \pm\,csc\,\theta$

三角函數在各象限的正或負

	第一象限	第二象限	第三象限	第四象限
sin θ	+	+	−	−
cos θ	+	−	−	+
tan θ	+	−	+	−

15. $sin(\frac{n\pi}{2} \pm \theta) = cos\,\theta$ $(n = 4k + 1$, $k \in z)$;

$sin(\frac{n\pi}{2} \pm \theta) = -\,cos\,\theta$ $((n = 4k + 3$, $k \in z)$

16. $cos(\frac{n\pi}{2} \pm \theta) = \mp sin\,\theta$ $((n = 4k + 1$, $k \in z)$;

$cos(\frac{n\pi}{2} \pm \theta) = \pm\, sin\,\theta$ $((n = 4k + 3$, $k \in z)$

17. $tan(\frac{n\pi}{2} \pm \theta) = \mp cot\,\theta$

18. $cot(\frac{n\pi}{2} \pm \theta) = \mp tan\,\theta$

19. $csc(\frac{n\pi}{2} \pm \theta) = sec\,\theta$ $((n = 4k + 1$, $k \in z)$;

$csc(\frac{n\pi}{2} \pm \theta) = -\,sec\,\theta$ $((n = 4k + 3$, $k \in z)$

20. $sec(\frac{n\pi}{2} \pm \theta) = \mp csc\,\theta$ $((n = 4k + 1$, $k \in z)$;

$sec(\frac{n\pi}{2} \pm \theta) = \pm\, csc\,\theta$ $((n = 4k + 3$, $k \in z)$

特例角的三角函數值

	$0°$	$30°$	$45°$	$60°$	$90°$
sin θ	**0**	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	**1**	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	**0**	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

21. $sin^2\,\theta + cos^2\,\theta = 1$
22. $1 + tan^2\,\theta = sec^2\,\theta$
23. $1 + cot^2\,\theta = csc^2\,\theta$
24. $tan\,\theta = \frac{sin\,\theta}{cos\,\theta}$
25. $cot\,\theta = \frac{cos\,\theta}{sin\,\theta}$

25-1. $sin(-\theta) = -sin\,\theta$
25-2. $cos(-\theta) = cos\,\theta$
25-3. $tan(-\theta) = -tan\,\theta$
25-4. $cot(-\theta) = -cot\,\theta$
25-5. $sec(-\theta) = sec\,\theta$
25-6. $csc(-\theta) = -csc\,\theta$

正弦定理(若 a、b、c 為△ABC 的三邊長)

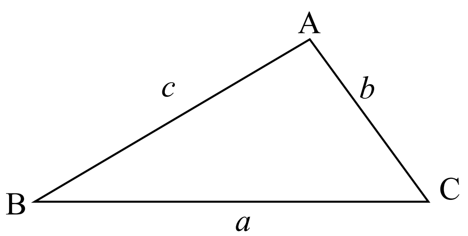
26. $\frac{a}{sin\,A} = \frac{b}{sin\,B} = \frac{c}{sin\,C} = 2R$

R 是△ABC 外接圓的半徑

餘弦定理(若 a、b、c 為△ABC 的三邊長)

27. $a^2 = b^2 + c^2 - 2bc\,cos\,A$
28. $b^2 = a^2 + c^2 - 2ac\,cos\,B$
29. $c^2 = a^2 + b^2 - 2ab\,cos\,C$
30. $cos\,A = \frac{b^2+c^2-a^2}{2bc}$
31. $cos\,B = \frac{a^2+c^2-b^2}{2ac}$
32. $cos\,C = \frac{a^2+b^2-c^2}{2ab}$

投影定理



33. $a = b\,cos\,C + c\,cos\,B$
34. $b = a\,cos\,C + c\,cos\,A$
35. $c = b\,cos\,A + a\,cos\,B$

加法定理

36. $sin(\alpha + \beta) = sin\,\alpha\,cos\,\beta + cos\,\alpha\,sin\,\beta$
37. $sin(\alpha - \beta) = sin\,\alpha\,cos\,\beta - cos\,\alpha\,sin\,\beta$
38. $cos(\alpha + \beta) = cos\,\alpha\,cos\,\beta - sin\,\alpha\,sin\,\beta$
39. $cos(\alpha - \beta) = cos\,\alpha\,cos\,\beta + sin\,\alpha\,sin\,\beta$
40. $tan(\alpha + \beta) = \frac{tan\,\alpha + tan\,\beta}{1 - tan\,\alpha\,tan\,\beta}$

41. $tan(\alpha-\beta)=\frac{tan\,\alpha-tan\,\beta}{1+tan\,\alpha\,tan\,\beta}$

42. $cot(\alpha+\beta)=\frac{cot\,\alpha\,cot\,\beta-1}{cot\,\beta+cot\,\alpha}$

43. $cot(\alpha-\beta)=\frac{cot\,\alpha\,cot\,\beta+1}{cot\,\beta-cot\,\alpha}$

44. $sec(\alpha+\beta)=\frac{sec\,\alpha\,sec\,\beta}{1-tan\,\alpha\,tan\,\beta}$

45. $sec(\alpha-\beta)=\frac{sec\,\alpha\,sec\,\beta}{1+tan\,\alpha\,tan\,\beta}$

46. $csc(\alpha+\beta)=\frac{csc\,\alpha\,csc\,\beta}{cot\,\varepsilon+cot\,\beta-1}$

47. $csc(\alpha-\beta)=\frac{csc\,\alpha\,csc\,\beta}{cot\,\alpha\,cot\,\beta+1}$

倍角定理

48. $sin\,2\,\theta=2\,sin\,\theta\,cos\,\theta$

49. $cos\,2\,\theta=cos^2\,\theta-sin^2\,\theta=2\,cos^2\,\theta-1=1-2\,sin^2\,\theta=\frac{1-tan^2\,\theta}{1+tan^2\,\theta}$

50. $tan\,2\,\theta=\frac{2\,tan\,\theta}{1-tan^2\,\theta}$

51. $cot\,2\,\theta=\frac{cot\,\theta-tan\,\theta}{2}=\frac{cot^2\,\theta-1}{2\,cot\,\theta}$

52. $sec\,2\,\theta=\frac{sec^2\,\theta}{1-tan^2\,\theta}=\frac{cot\,\theta+tan\,\theta}{cot\,\theta-tan\,\theta}$

53. $csc\,2\,\theta=\frac{sec\,\theta\,csc\,\theta}{2}=\frac{cot\,\theta+tan\,\theta}{2}$

54. $sin\,3\,\theta=-4\,sin^3\,\theta+3\,sin\,\theta$

55. $cos\,3\,\theta=4\,cos^3\,\theta-3\,cos\,\theta$

56. $tan\,3\,\theta=\frac{3\,tan\,\theta-tan^3\,\theta}{1-3\,tan^2\,\theta}$

57. $cot\,3\,\theta=\frac{cot^3\,\theta-3\,cot\,\theta}{3\,cot^2\,\theta-1}$

58. $sec\,3\,\theta=\frac{sec^3\,\theta}{4-3\,sec^2\,\theta}$

59. $csc\,3\,\theta=\frac{csc^3\,\theta}{3\,csc^2\,\theta-4}$

半角公式

60. $sin\frac{\theta}{2} = \pm\sqrt{\frac{1-cos\,\theta}{2}}$

61. $cos\frac{\theta}{2} = \pm\sqrt{\frac{1+cos\,\theta}{2}}$

62. $tan\frac{\theta}{2} = \pm\sqrt{\frac{1-cos\,\theta}{1+cos\,\theta}} = \pm\frac{sin\,\theta}{1+cos\,\theta} = \pm\frac{1-cos\,\theta}{sin\,\theta} = \pm(csc\,\theta - cot\,\theta)$

63. $cot\frac{\theta}{2} = \pm\sqrt{\frac{1+cos\,\theta}{1-cos\,\theta}} = \pm\frac{sin\,\theta}{1-cos\,\theta} = \pm\frac{1+cos\,\theta}{sin\,\theta} = \pm(csc\,\theta + cot\,\theta)$

積化和差公式

64. $sin\,\alpha\,cos\,\beta = \frac{sin(\alpha+\beta)+sin(\alpha-\beta)}{2}$

65. $cos\,\alpha\,sin\,\beta = \frac{sin(\alpha+\beta)-sin(\alpha-\beta)}{2}$

66. $cos\,\alpha\,cos\,\beta = \frac{cos(\alpha+\beta)+cos(\alpha-\beta)}{2}$

67. $sin\,\alpha\,sin\,\beta = \frac{-cos(\alpha+\beta)+cos(\alpha-\beta)}{2}$

和差化積公式

68. $sin\,\alpha + sin\,\beta = 2\,sin\frac{\alpha+\beta}{2}cos\frac{\alpha-\beta}{2}$

69. $sin\,\alpha - sin\,\beta = 2\,cos\frac{\alpha+\beta}{2}sin\frac{\alpha-\beta}{2}$

70. $cos\,\alpha + cos\,\beta = 2\,cos\frac{\alpha+\beta}{2}cos\frac{\alpha-\beta}{2}$

71. $cos\,\alpha - cos\,\beta = -2\,sin\frac{\alpha+\beta}{2}sin\frac{\alpha-\beta}{2}$