

(71)乘法公式

我們首先列舉所有的乘法公式：

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)^2 = (a - b)^2 + 4ab$
4. $(a - b)^2 = (a + b)^2 - 4ab$
5. $(a + b)(a - b) = a^2 - b^2$
6. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
7. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
8. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
9. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
10. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
11. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
12. $a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$
13. $\sqrt{a + b + 2\sqrt{ab}} = \sqrt{(\sqrt{a} + \sqrt{b})^2} = \sqrt{a} + \sqrt{b}$
14. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
15. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

$$1. \quad (0.999)^2 = (1 - 0.001)^2 = 1 - 2 \times 0.001 + (0.001)^2 = 0.998 + 0.000001 = 0.998001$$

$$2. \quad (1.17)^2 = (1 + 0.17)^2 = 1 + 2 \times 0.17 + (0.17)^2 = 1 + 0.34 + 0.0289 = 1.34 + 0.0289 = 1.3689$$

$$3. \quad \text{已知 } x+y=5, xy=6, \text{ 求 } x^2 + y^2$$

$$x^2 + y^2 = (x + y)^2 - 2xy = 5^2 - 2 \times 6 = 25 - 12 = 13$$

這題也可不必使用乘法公式來計算

$$\because xy = 6 \therefore x = \frac{6}{y}$$

$$\because x + y = \frac{6}{y} + y = \frac{y^2 + 6}{y} = 5$$

$$\therefore y^2 - 5y + 6 = 0$$

$$(y-2)(y-3)=0$$

$$y=2 \text{ 或 } y=3$$

$$\therefore x=3 \text{ 或 } x=2$$

$$\text{如 } x=2, y=3$$

$$x^2 + y^2 = 4 + 9 = 13$$

$$\text{如 } x=3, y=2$$

$$x^2 + y^2 = 9 + 4 = 13$$

同樣的答案

$$4. \quad \text{已知 } ac+bc=7, \text{ 求 } (a+b+c)^2 - (a+b-c)^2$$

$$(a+b+c)^2 - (a+b-c)^2$$

$$= (a+b+c+a+b-c)(a+b+c-a-b+c)$$

$$= 2(a+b)(2c)$$

$$= 4(a+b)c$$

$$= 4 \times 7$$

$$= 28$$

$$5. \quad x^2 + y^2 + z^2 - xy - yz - zx$$

$$= \frac{1}{2}(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$$

$$= \frac{1}{2}((x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2zx + x^2))$$

$$= \frac{1}{2}((x-y)^2 + (y-z)^2 + (z-x)^2)$$

$$\begin{aligned}
6. \quad & (2001) \times (2002) - (1999) \times (2004) \\
&= (2 \times 10^3 + 1)(2 \times 10^3 + 2) - (2 \times 10^3 - 1)(2 \times 10^3 + 4) \\
&= 4 \times 10^6 + 2 \times 2 \times 10^3 + 1 \times 2 \times 10^3 + 2 \\
&\quad - 4 \times 10^6 - 2 \times 4 \times 10^3 + 1 \times 2 \times 10^3 + 4 \\
&= 4 \times 10^6 + 4 \times 10^3 + 2 \times 10^3 + 2 - 4 \times 10^6 - 8 \times 10^3 + 2 \times 10^3 + 4 \\
&= 6 \times 10^3 + 2 - 6 \times 10^3 + 4 \\
&= 2 + 4 \\
&= 6
\end{aligned}$$

$$\begin{aligned}
7. \quad & \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{10^2}\right) \\
&= \left(1 + \frac{1}{2}\right) \left(1 - \frac{1}{2}\right) \left(1 + \frac{1}{3}\right) \left(1 - \frac{1}{3}\right) \cdots \left(1 + \frac{1}{10}\right) \left(1 - \frac{1}{10}\right) \\
&= \left(1 + \frac{1}{2}\right) \left(1 + \frac{1}{3}\right) \cdots \left(1 + \frac{1}{10}\right) \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \cdots \left(1 - \frac{1}{10}\right) \\
&= \left(\frac{3}{2} \times \frac{4}{3} \times \cdots \frac{10}{9} \times \frac{11}{10}\right) \left(\frac{1}{2} \times \frac{2}{3} \times \cdots \frac{8}{9} \times \frac{9}{10}\right) \\
&= \left(\frac{11}{2}\right) \left(\frac{1}{10}\right) \\
&= \frac{11}{20}
\end{aligned}$$

$$\begin{aligned}
8. \quad & x^2 - 2x - 399 \\
&= x^2 - 2x + 1 - 1 - 399 \\
&= (x - 1)^2 - 400 \\
&= (x - 1)^2 - 20^2 \\
&= (x - 1 + 20)(x - 1 - 20) \\
&= (x + 19)(x - 21)
\end{aligned}$$

$$\begin{aligned}
9. \quad & x^4 + x^2y^2 + y^4 \\
&= x^4 + 2x^2y^2 + y^4 - x^2y^2 \\
&= (x^2 + y^2)^2 - x^2y^2 \\
&= (x^2 + y^2)^2 - (xy)^2 \\
&= (x^2 + y^2 + xy)(x^2 + y^2 - xy)
\end{aligned}$$

$$\begin{aligned}
10. \quad & x^4 - 3x^2y^2 + y^4 \\
&= x^4 - 2x^2y^2 + y^4 - x^2y^2 \\
&= (x^2 - y^2)^2 - x^2y^2 \\
&= (x^2 - y^2)^2 - (xy)^2 \\
&= (x^2 - y^2 + xy)(x^2 - y^2 - xy)
\end{aligned}$$

$$\begin{aligned}
11. \quad & a^2 + b^2 - 2ab - 5a + 5b + 6 \\
&= (a - b)^2 - 5(a - b) + 6 \\
&= (a - b - 2)(a - b - 3)
\end{aligned}$$

$$\begin{aligned}
12. \quad & 9x^4 - 7x^2 + 1 \\
&= 9x^4 - 6x^2 + 1 - x^2 \\
&= (3x^2 - 1)^2 - x^2 \\
&= (3x^2 - 1 + x)(3x^2 - 1 - x) \\
&= (3x^2 + x - 1)(3x^2 - x - 1)
\end{aligned}$$

$$\begin{aligned}
13. \quad & (x - 1)(x - 3)(x - 5)(x - 7) + 15 \\
&= ((x - 1)(x - 7))((x - 3)(x - 5)) + 15 \\
&= (x^2 - 8x + 7)(x^2 - 8x + 15) + 15 \\
&= (x^2 - 8x)^2 + 22(x^2 - 8x) + 105 + 15 \\
&= (x^2 - 8x)^2 + 22(x^2 - 8x) + 120 \\
&= (x^2 - 8x + 10)(x^2 - 8x + 12)
\end{aligned}$$

$$14. \quad \text{已知 } x + \frac{1}{x} = 3, \text{ 求 } x^2 + \frac{1}{x^2}$$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2\left(x\right)\left(\frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^2 - 2 = 3^2 - 2 = 7$$

$$15. \quad \text{已知 } (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3, \text{ 求證 } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\text{證明} \quad \because (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\begin{aligned}
\therefore a^3 + b^3 &= (a + b)^3 - 3a^2b - 3ab^2 \\
&= (a + b)^3 - 3ab(a + b) \\
&= (a + b)((a + b)^2 - 3ab) \\
&= (a + b)(a^2 - ab + b^2)
\end{aligned}$$

$$\text{同理可證 } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

16. 求證 $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right)$

證明:

令 $x=a, \frac{1}{x}=b$

$\therefore (a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

$\therefore \left(x^3 + \frac{1}{x^3}\right) = \left(x + \frac{1}{x}\right)\left(x^2 - x \cdot \frac{1}{x} + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$

17. 承上題，已知 $x + \frac{1}{x} = 3$ ，求 $x^3 + \frac{1}{x^3}$

$\left(x^3 + \frac{1}{x^3}\right) = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$

從 14 題得知 $x^2 + \frac{1}{x^2} = 7$

故 $\left(x^3 + \frac{1}{x^3}\right) = 3(7 - 1) = 3 \times 6 = 18$

18. $6(x + y)^2 - 5(x^2 - y^2) + (x - y)^2$
 $= (3(x + y) - (x - y))(2(x + y) - (x - y))$
 $= (2x + 4y)(x + 3y)$
 $= 2(x + 2y)(x + 3y)$

19. $(x + 1)(x + 2)(x + 3)(x + 4) - 63$
 $= ((x + 1)(x + 4))((x + 2)(x + 3)) - 63$
 $= (x^2 + 5x + 4)(x^2 + 5x + 6) - 63$
 $= (x^2 + 5x)^2 + 10(x^2 + 5x) + 24 - 63$
 $= (x^2 + 5x)^2 + 10(x^2 + 5x) - 39$
 $= (x^2 + 5x + 13)(x^2 + 5x - 3)$

20. 已知 $(2x+3)$ 為 $8x^2 + ax + 9$ 的因式，求 a
 $(2x + 3)(4x + 3) = 8x^2 + 18x + 9$
 故 $a=18$

$$\begin{aligned}
21. & \text{求}(2+1)(2^2+1)(2^4+1)(2^8+1) \\
& = (2-1)(2+1)(2^2+1)(2^4+1)(2^8+1) \\
& = (2^2-1)(2^2+1)(2^4+1)(2^8+1) \\
& = (2^4-1)(2^4+1)(2^8+1) \\
& = (2^8-1)(2^8+1) \\
& = 2^{16}-1
\end{aligned}$$

$$\text{可得}(2+1)(2^2+1)(2^4+1)(2^8+1)\cdots(2^n+1)=2^{2n}-1$$

$$\begin{aligned}
22. & (3x+2)(-x^6+3x^5)+(3x+2)(-2x^6+x^5)+(x+1)(3x^6-4x^5) \\
& = (3x+2)(-x^6+3x^5-2x^6+x^5)+(x+1)(3x^6-4x^5) \\
& = (3x+2)(-3x^6+4x^5)+(x+1)(3x^6-4x^5) \\
& = (3x^6-4x^5)(-3x-2+x+1) \\
& = (3x^6-4x^5)(-2x-1) \\
& = -(2x+1)(3x^6-4x^5)
\end{aligned}$$

$$\begin{aligned}
23. & \text{已知 } a+b=8, ab=15, \text{ 求 } a^2-b^2, a^2+b^2, a-b \\
& a^2+2ab+b^2=(a+b)^2 \\
& \therefore a^2+b^2=(a+b)^2-2ab=8^2-2\times 15=64-30=34 \\
& (a-b)^2=a^2+b^2-2ab=34-2\times 15=34-30=4 \\
& \therefore a-b=\pm 2 \\
& \text{若 } a-b=2 \quad a^2-b^2=(a+b)(a-b)=8\times 2=16
\end{aligned}$$

$$a-b=-2 \quad a^2-b^2=(a+b)(a-b)=8\times (-2)=-16$$

$$\begin{aligned}
24. & \frac{1987^2-2\times 1987\times 7+7^2}{1980}\times \frac{1994}{1987^2-7^2} \\
& = \frac{(1987-7)^2}{1980}\times \frac{1994}{(1987+7)(1987-7)} \\
& = \frac{1980^2}{1980}\times \frac{1994}{(1994)(1980)} \\
& = 1980\times \frac{1}{1980} \\
& = 1
\end{aligned}$$

$$\begin{aligned}
25. \quad & (2x-3y)^2(2x+3y)^2 - (3x+2y)^2(3x-2y)^2 \\
&= (4x^2-9y^2)^2 - (9x^2-4y^2)^2 \\
&= (4x^2-9y^2+9x^2-4y^2)(4x^2-9y^2-9x^2+4y^2) \\
&= (13x^2-13y^2)(-5x^2-5y^2) \\
&= 13(x^2-y^2)(-5)(x^2+y^2) \\
&= -65(x^2-y^2)(x^2+y^2) \\
&= -65(x+y)(x-y)(x^2+y^2)
\end{aligned}$$

26. 已知 $x+y=4, xy=1$, 求 $\frac{y}{x+1} + \frac{x}{y+1}$

$$\begin{aligned}
& \frac{y}{x+1} + \frac{x}{y+1} \\
&= \frac{y(y+1) + x(x+1)}{(x+1)(y+1)} \\
&= \frac{x^2 + y^2 + x + y}{(x+1)(y+1)} \\
&= \frac{x^2 + y^2 + 2xy - 2xy + (x+y)}{xy + x + y + 1} \\
&= \frac{(x+y)^2 - 2xy + (x+y)}{(x+y) + xy + 1} \\
&= \frac{4^2 - 2 + 4}{4 + 1 + 1} \\
&= \frac{18}{6} \\
&= 3
\end{aligned}$$

27. 求 $\frac{\sqrt{54}}{3} + \frac{3}{\sqrt{6}} - \frac{8}{\sqrt{24}}$

$$\begin{aligned} & \frac{\sqrt{54}}{3} + \frac{3}{\sqrt{6}} - \frac{8}{\sqrt{24}} \\ &= \frac{\sqrt{9 \times 6}}{3} + \frac{3}{\sqrt{6}} - \frac{8}{\sqrt{4 \times 6}} \\ &= \frac{3\sqrt{6}}{3} + \frac{3}{\sqrt{6}} - \frac{8}{2\sqrt{6}} \\ &= \sqrt{6} + \frac{3-4}{\sqrt{6}} \\ &= \sqrt{6} - \frac{1}{\sqrt{6}} \\ &= \frac{6-1}{\sqrt{6}} \\ &= \frac{5}{\sqrt{6}} \end{aligned}$$

28. 求 $(1 - \sqrt[3]{2})(1 + \sqrt[3]{2} + \sqrt[3]{4})$

令 $a=1, b=\sqrt[3]{2}$

利用 $(a-b)(a^2+ab+b^2) = a^3-b^3$

故 $(1 - \sqrt[3]{2})(1 + \sqrt[3]{2} + \sqrt[3]{4}) = 1^3 - (\sqrt[3]{2})^3 = 1 - 2 = -1$

29. 分解因式 $1 + 8x^3$

根據 $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$1 + 8x^3 = (1 + 2x)(1 - 1(2x) + (2x)^2) = (1 + 2x)(1 - 2x + 4x^2)$

30. 化簡 $\sqrt{13 + 4\sqrt{10}}$

根據 $\sqrt{a+b+2\sqrt{ab}} = \sqrt{a} + \sqrt{b}$

$\therefore \sqrt{13 + 4\sqrt{10}} = \sqrt{8} + \sqrt{5} = 2\sqrt{2} + \sqrt{5}$

31. $a = \sqrt{8} + \sqrt{3}$, $b = \sqrt{6} + \sqrt{5}$ 比較大小

$$a^2 = 8 + 3 + 2\sqrt{8 \times 3} = 11 + 2\sqrt{24}$$

$$b^2 = 6 + 5 + 2\sqrt{6 \times 5} = 11 + 2\sqrt{30}$$

$$b^2 > a^2$$

故 $b > a$

32. $a = \frac{\sqrt{3}+1}{\sqrt{3}-1}$, $b = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, 求 $a^3 + b^3$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a + b = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} + \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \frac{(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{3 + 2\sqrt{3} + 1 + 3 - 2\sqrt{3} + 1}{3 - 1}$$

$$= \frac{4 + 4}{2}$$

$$= \frac{8}{2}$$

$$= 4$$

$$ab = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 1$$

$$a^2 - ab + b^2 = (a + b)^2 - 3ab = 4^2 - 3 = 16 - 3 = 13$$

$$a^3 + b^3 = 4 \times 13 = 52$$

33. 化簡 $(2 - x)(2 + x)(4 + x^2)(16 + x^4)$

$$= (4 - x^2)(4 + x^2)(16 + x^4)$$

$$= (4^2 - (x^2)^2)(16 + x^4)$$

$$= (16 - x^4)(16 + x^4)$$

$$= 16^2 - (x^4)^2$$

$$= 256 - x^8$$

34. 利用平方差化簡 $(x-2)(x+2)(x^2+2x+4)(x^2-2x+4)$

$$(x-2)(x+2)(x^2+2x+4)(x^2-2x+4)$$

$$= (x^2-4)((x^2+4)^2-(2x)^2)$$

$$= (x^2-4)(x^4+8x^2+16-4x^2)$$

$$= (x^2-4)(x^4+4x^2+16)$$

35. 化簡 $\sqrt{10+4\sqrt{3-2\sqrt{2}}}$

$$\text{利用 } \sqrt{a+b+2\sqrt{ab}} = \sqrt{a} + \sqrt{b}$$

$$\sqrt{10+4\sqrt{3-2\sqrt{2}}}$$

$$= \sqrt{10+4(\sqrt{2}-1)}$$

$$= \sqrt{6+4\sqrt{2}}$$

$$= \sqrt{2} + 2$$

36. 試證 $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{n^2}\right)$$

$$= \left(1-\frac{1}{2}\right)\left(1+\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\cdots\left(1-\frac{1}{n}\right)\left(1+\frac{1}{n}\right)$$

$$= \frac{1}{2} \times \frac{3}{2} \times \frac{2}{3} \times \frac{4}{3} \times \frac{3}{4} \times \frac{5}{4} \times \cdots \frac{n-1}{n} \times \frac{n+1}{n}$$

$$= \frac{1}{2} \times \frac{n+1}{n}$$

$$= \frac{n+1}{2n}$$

37. 化簡 $(1 + \sqrt{3} + \sqrt{5})(1 + \sqrt{3} - \sqrt{5})$

$$\begin{aligned} & (1 + \sqrt{3} + \sqrt{5})(1 + \sqrt{3} - \sqrt{5}) \\ &= (1 + \sqrt{3})^2 - (\sqrt{5})^2 \\ &= 1 + 2\sqrt{3} + 3 - 5 \\ &= -1 + 2\sqrt{3} \end{aligned}$$

38. 化簡 $((1 + \sqrt{3})^2 - 5)(5 - (1 - \sqrt{3})^2)$

$$\begin{aligned} & ((1 + \sqrt{3})^2 - 5)(5 - (1 - \sqrt{3})^2) \\ &= (1 + 2\sqrt{3} + 3 - 5)(5 - (1 - 2\sqrt{3} + 3)) \\ &= (-1 + 2\sqrt{3})(1 + 2\sqrt{3}) \\ &= (2\sqrt{3} + 1)(2\sqrt{3} - 1) \\ &= ((2\sqrt{3})^2 - 1) \\ &= 12 - 1 \\ &= 11 \end{aligned}$$

39. 化簡 $a^2b + ab^2 - ab - a - b + 1$

$$\begin{aligned} & a^2b + ab^2 - ab - a - b + 1 \\ &= ab(a + b) - ab - (a + b) + 1 \\ &= (a + b)(ab - 1) - ab + 1 \\ &= (a + b)(ab - 1) - (ab - 1) \\ &= (ab - 1)(a + b - 1) \end{aligned}$$

40. 因式分解 $(x^2 + y^2 - 4)^2 - 4x^2y^2$

$$\begin{aligned} & (x^2 + y^2 - 4)^2 - 4x^2y^2 \\ &= (x^2 + y^2 - 4 + 2xy)(x^2 + y^2 - 4 - 2xy) \\ &= (x^2 + y^2 + 2xy - 4)(x^2 + y^2 - 2xy - 4) \\ &= ((x + y)^2 - 4)((x - y)^2 - 4) \\ &= (x + y + 2)(x + y - 2)(x - y + 2)(x - y - 2) \end{aligned}$$

41. 因式分解 $a^2 + b^2 - 2ab - 5a + 5b + 6$

$$\begin{aligned} & a^2 + b^2 - 2ab - 5a + 5b + 6 \\ &= (a - b)^2 - 5(a - b) + 6 \\ &= (a - b - 2)(a - b - 3) \end{aligned}$$

42. 化簡 $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{9}+\sqrt{10}}$

$$\begin{aligned} & \frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{9}+\sqrt{10}} \\ &= \frac{1-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3} + \frac{\sqrt{3}-\sqrt{4}}{3-4} + \cdots + \frac{\sqrt{9}-\sqrt{10}}{9-10} \\ &= \frac{-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} + \frac{\sqrt{3}-\sqrt{4}}{-1} + \cdots + \frac{\sqrt{9}-\sqrt{10}}{-1} \\ &= \sqrt{2} + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \cdots + \sqrt{10} - \sqrt{9} \\ &= \sqrt{10} \end{aligned}$$

43. 化簡 $(x^2 + 2x + 4)(x^2 - 2x + 4)$

$$\begin{aligned} & (x^2 + 2x + 4)(x^2 - 2x + 4) \\ &= (x^2 + 4 + 2x)(x^2 + 4 - 2x) \\ &= (x^2 + 4)^2 - (2x)^2 \\ &= x^4 + 8x^2 + 16 - 4x^2 \\ &= x^4 + 4x^2 + 16 \end{aligned}$$

44. 分解因式 $x^6 - 2^6$

$$\begin{aligned} & x^6 - 2^6 \\ &= (x^3)^2 - (2^3)^2 \\ &= (x^3 + 2^3)(x^3 - 2^3) \\ &= (x + 2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4) \\ &= (x + 2)(x - 2)(x^2 - 2x + 4)(x^2 + 2x + 4) \end{aligned}$$

45. 已知 $6x^2 - 7x - 3 = (ax + 1)(2x + b)$, 求 $a + b$

$$\begin{aligned} & (ax + 1)(2x + b) = 2ax^2 + abx + 2x + b = 6x^2 - 7x - 3 \\ & 2a = 6, a = 3 \\ & b = -3 \\ & a + b = 3 - 3 = 0 \end{aligned}$$

46. 已知 $5x^2 + 17x - 12 = (x + a)(bx + c)$, 求 a, b, c

$$\begin{aligned} & 5x^2 + 17x - 12 = (x + 4)(5x - 3) \\ & a = 4, b = 5, c = -3 \end{aligned}$$

$$\begin{aligned}
47. & (3x+2)(-x^6+3x^5) + (3x+2)(-2x^6+x^5) + (x+1)(3x^6-4x^5) \\
&= (3x+2)(-x^6+3x^5-2x^6+x^5) + (x+1)(3x^6-4x^5) \\
&= (3x+2)(-3x^6+4x^5) + (x+1)(3x^6-4x^5) \\
&= (x+1)(3x^6-4x^5) - (3x+2)(3x^6-4x^5) \\
&= (3x^6-4x^5)(x+1-3x-2) \\
&= (3x^6-4x^5)(-2x-1)
\end{aligned}$$

$$\begin{aligned}
48. & x^4 + 64 \\
& x^4 + 64 = (x^2 + 8)^2 - 16x^2 = (x^2 + 8 + 4x)(x^2 + 8 - 4x) \\
&= (x^2 + 4x + 8)(x^2 - 4x + 8)
\end{aligned}$$

$$\begin{aligned}
49. & 6(x+y)^2 + 5(x^2 - y^2) + (x-y)^2 \\
&= (3(x+y) + (x-y))(2(x+y) + (x-y)) \\
&= (4x+2y)(3x+y) \\
&= 2(2x+y)(3x+y)
\end{aligned}$$

$$\begin{aligned}
50. & 1 \times 3 \times 5 \times 17 \times 257 \\
& 1 \times 3 \times 5 \times 17 \times 257 \\
&= (2-1)(2+1)(2^2+1)(2^4+1)(2^8+1) \\
&= (2^2-1)(2^2+1)(2^4+1)(2^8+1) \\
&= (2^4-1)(2^4+1)(2^8+1) \\
&= (2^8-1)(2^8+1) \\
&= 2^{16} - 1 \\
&= 65536 - 1 \\
&= 65535
\end{aligned}$$

$$\begin{aligned}
51. & 1001^2 - 2002 \\
&= 1001^2 - 2 \times 1001 \\
&= (1001)(1001-2) \\
&= (1001)(999) \\
&= (1000+1)(1000-1) \\
&= 1000^2 - 1 \\
&= 999999
\end{aligned}$$

$$\begin{aligned}
52. & 399^2 = a + 1, \text{ 求 } a \\
& a = 399^2 - 1 = (399+1)(399-1) = (400)(398) = 159200
\end{aligned}$$

53. 求 99^2

$$99^2 - 1 = (99 + 1)(99 - 1) = (100)(98) = 9800$$

$$99^2 = 9800 + 1 = 9801$$

54. 承上題，還有一種做法

$$99^2 = (100 - 1)^2 = 10000 - 200 + 1 = 9801$$

同樣的結果

55. $9 \times 11 \times 101 \times (10^4 + 1) \times (10^8 + 1)$

$$= (10 - 1)(10 + 1)(10^2 + 1)(10^4 + 1)(10^8 + 1)$$

$$= (10^2 - 1)(10^2 + 1)(10^4 + 1)(10^8 + 1)$$

$$= (10^4 - 1)(10^4 + 1)(10^8 + 1)$$

$$= (10^8 - 1)(10^8 + 1)$$

$$= 10^{16} - 1$$

56. 已知 $a + b = 5$ ， $ab = 4$ ，求 $a^2 + b^2$ ， $3a^2 + 3b^2 - 2ab$ ， $(a - b)^2$

$$a^2 + b^2 = (a + b)^2 - 2ab = 5^2 - 2 \times 4 = 25 - 8 = 17$$

$$3a^2 + 3b^2 - 2ab = 3(a^2 + b^2) - 2 \times 4 = 3(17) - 8 = 51 - 8 = 43$$

$$(a - b)^2 = a^2 - 2ab + b^2 = 17 - 2(4) = 9$$

57. $2001 \times 2002 - 1999 \times 2004$

$$= (2001)(2001 + 1) - (2001 - 2)(2001 + 3)$$

$$= (2001)^2 + 2001 - (2001^2 + 2001 - 6)$$

$$= 6$$

58. 503×497

$$= (500 + 3) \times (500 - 3)$$

$$= (500)^2 - 3^2$$

$$= 250000 - 9$$

$$= 249991$$

59. 9×199

$$= (10 - 1) \times (200 - 1)$$

$$= (10 - 1) \times (2 \times 100 - 1)$$

$$= 10 \times 2 \times 100 - 10 - 2 \times 100 + 1$$

$$= 2000 - 200 - 9$$

$$= 1800 - 9$$

$$= 1791$$

$$\begin{aligned}
60. \quad & x^2 - 2x - 399 \\
&= x^2 - 2x + 1 - 1 - 399 \\
&= (x - 1)^2 - 20^2 \\
&= (x - 1 + 20)(x - 1 - 20) \\
&= (x + 19)(x - 21)
\end{aligned}$$