**(30) 指數練習題**

1. $(3^{-\frac{1}{2}})^{4}=3^{-\frac{1}{2}×4}=3^{-2}=\frac{1}{3^{2}}=\frac{1}{9}$

2. $(3^{\frac{5}{3}})^{\frac{2}{5}}=3^{\frac{5}{3}×\frac{2}{5}}=3^{\frac{2}{3}}=\sqrt[3]{3^{2}}=\sqrt[3]{9}$

3. $5^{\frac{3}{2}}×5=5^{\frac{3}{2}+1}=5^{\frac{5}{2}}=5∙5^{\frac{5-2}{2}}=5∙5×5^{\frac{1}{2}}=25\sqrt{5}$

4. $4^{\frac{3}{2}}÷4^{\frac{1}{2}}=4^{\frac{3}{2}-\frac{1}{2}}=4^{\frac{2}{2}}=4^{1}=4$

5.$ (3^{\frac{1}{2}})^{3}=3^{\frac{3}{2}}=3×3^{\frac{3-2}{2}}=3×3^{\frac{1}{2}}=3\sqrt{3}$

6.$ (4^{\frac{1}{2}})^{-1}=4^{-\frac{1}{2}}=\frac{1}{4^{\frac{1}{2}}}=\frac{1}{2}$

7.$(4^{\frac{1}{3}})(4^{\frac{2}{3}})=4^{\frac{1}{3}+\frac{2}{3}}=4^{\frac{3}{3}}=4^{1}=4$

8.$ 2^{\frac{1}{2}}×3^{\frac{1}{2}}=(2×3)^{\frac{1}{2}}=6^{\frac{1}{2}}=\sqrt{6}$

9.$ 3^{\frac{3}{2}}×4^{\frac{3}{2}}=12^{\frac{3}{2}}=12∙12^{\frac{3-1}{2}}=12∙12^{\frac{1}{2}}=12\sqrt{12}=12×\sqrt{4×3}=12×2\sqrt{3}=24\sqrt{3}$

10.$ (-2)^{\frac{1}{2}}×(-3)^{\frac{1}{2}}=(6)^{\frac{1}{2}}=\sqrt{6}$

11.$ 3^{\frac{1}{3}}×9^{\frac{1}{3}}=(3×9)^{\frac{1}{3}}=(27)^{\frac{1}{3}}=3$

12.$ 125^{\frac{1}{3}}×5=\sqrt[3]{125}×5=5×5=25$

13.$ 2^{\frac{3}{2}}×4^{\frac{3}{2}}=8^{\frac{3}{2}}=8×8^{\frac{3-2}{2}}=8×8^{\frac{1}{2}}=8\sqrt{8}=8\sqrt{4×2}=8×2\sqrt{2}=16\sqrt{2}$

14.$ 5^{\frac{3}{2}}×(\frac{2}{5})^{\frac{3}{2}}=(5×\frac{2}{5})^{\frac{3}{2}}=2^{\frac{3}{2}}=2×2^{\frac{1}{2}}=2\sqrt{2}$

15.$ (\frac{1}{5})^{\frac{1}{2}}×(125)^{\frac{1}{2}}=(\frac{1}{5}×125)^{\frac{1}{2}}=(25)^{\frac{1}{2}}=5$

16.$ (\frac{1}{5})^{\frac{3}{2}}×(\frac{1}{4})^{\frac{3}{2}}=(\frac{1}{20})^{\frac{3}{2}}=\frac{1}{(20^{\frac{3}{2}})}=\frac{1}{20×20^{\frac{3-2}{2}}}=\frac{1}{20×\sqrt{20}}=\frac{1}{20×\sqrt{4×5}}=\frac{1}{20×2\sqrt{5}}=\frac{1}{40\sqrt{5}}$

17.$ 3^{\frac{1}{2}}×\left(27\right)^{\frac{1}{2}}=\left(3×27\right)^{\frac{1}{2}}=\left(81\right)^{\frac{1}{2}}=9$

18.$ (5^{\frac{3}{5}})^{\frac{10}{3}}×5^{-2}=5^{\frac{3}{5}×\frac{10}{3}}×5^{-2}=5^{2}×5^{-2}=5^{0}=1$

19.$\frac{a^{2}b^{\frac{1}{2}}}{ab}=ab^{\frac{1}{2}-1}=ab^{-\frac{1}{2}}=\frac{a}{\sqrt{b}}$

20.$(a+b)^{-2}=\frac{1}{(a+b)^{2}}$

21.$ (a+b)^{-2}c^{2}=\frac{c^{2}}{(a+b)^{2}}=(\frac{c}{a+b})^{2}$

22.$\left(a-a^{-1}\right)a=a^{2}-1$

23.$ (a+a^{-1})\left(a-a^{-1}\right)=a^{2}-(a^{-1})^{2}=a^{2}-a^{-2}=a^{2}-\frac{1}{a^{2}}=\frac{a^{4}-1}{a^{2}}$

24.$ (a+a^{-1})(a)\left(a-1\right)^{-1}=(a×a+a^{-1}×a)\left(a-1\right)^{-1}=(a^{2}-1)\left(a-1\right)^{-1}=(a+1)(a-1)\left(a-1\right)^{-1}=(a+1)\left(a-1\right)^{1-1}=(a+1)$

25.$(a^{2}-b^{2})\left(a+b\right)^{-1}=(a+b)(a-b)\left(a+b\right)^{-1}=(a+b)\left(a+b\right)^{-1}(a-b)=\left(a+b\right)^{1-1}(a-b)=(a-b)$

26.$(a^{\frac{1}{2}}+a^{\frac{3}{2}})a^{\frac{1}{2}}=a^{\frac{1}{2}+\frac{1}{2}}+a^{\frac{3}{2}+\frac{1}{2}}=a+a^{2}=a(1+a)$

27.$a^{\frac{3}{2}}b^{-\frac{1}{2}}+a^{\frac{1}{2}}b^{\frac{1}{2}}=a^{\frac{1}{2}}b^{-\frac{1}{2}}(a+b)$

28.$ a^{\frac{3}{2}}b^{\frac{1}{2}}+a^{\frac{1}{2}}b^{\frac{3}{2}}=a^{\frac{1}{2}}b^{\frac{1}{2}}(a+b)$

29.$(a^{\frac{1}{2}}+b^{\frac{1}{2}})(a-b)=a^{\frac{1}{2}}a-a^{\frac{1}{2}}b+b^{\frac{1}{2}}a-b^{\frac{1}{2}}b=a^{\frac{3}{2}}-a^{\frac{1}{2}}b+b^{\frac{1}{2}}a-b^{\frac{3}{2}}$

30.$a^{-1}b^{-2}+a^{-2}b^{-1}=a^{-2}b^{-2}\left(a+b\right)=\frac{a+b}{a^{2}b^{2}}$

31.解方程式

$x^{2}-(\sqrt{a}+\sqrt{b})x+\sqrt{ab}$=0

$$(x-\sqrt{a})(x-\sqrt{b})=0$$

$$∴x=\sqrt{a}$$

$$x=\sqrt{b}$$

32. 解方程式

$$\sqrt{a}x+\sqrt{b}x=\sqrt{c}$$

$$x(\sqrt{a}+\sqrt{b})=\sqrt{c}$$

$$x=\frac{\sqrt{c}}{\sqrt{a}+\sqrt{b}}=\frac{\sqrt{c}(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})}=\frac{\sqrt{ac}-\sqrt{bc}}{a-b}$$

33.$x^{2}-3\sqrt{2}x+4=0$

$$\left(x-2\sqrt{2}\right)\left(x-\sqrt{2}\right)=0$$

$$∴x=2\sqrt{2}$$

$$x=\sqrt{2}$$

也可以代公式 $x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$

a=1

$$b=-3\sqrt{2}$$

c=4

$$∴x=\frac{3\sqrt{2}\pm \sqrt{(-3\sqrt{2})^{2}-4(1)(4)}}{2}=\frac{3\sqrt{2}\pm \sqrt{18-16}}{2}=\frac{3\sqrt{2}\pm \sqrt{2}}{2}$$

$$x=\frac{3\sqrt{2}+\sqrt{2}}{2}=\frac{4\sqrt{2}}{2}=2\sqrt{2}$$

$$x=\frac{3\sqrt{2}-\sqrt{2}}{2}=\frac{2\sqrt{2}}{2}\sqrt{2}$$

34.$x^{2}-\left(a^{-1}+2a\right)x+2=0$

$$(x-a^{-1})(x-2a)=0$$

$$x=a^{-1}=\frac{1}{a}$$

x=2a

35.$ x^{2}-\left(a^{-1}+a\right)x+1=0$

$$(x-a^{-1})(x-a)=0$$

$$x=a^{-1}=\frac{1}{a}$$

x=a

36.$2x^{2}+2\sqrt{2}x+1=0$

$$(\sqrt{2}x+1)^{2}=0$$

$$\sqrt{2}x+1=0$$

$$x=-\frac{1}{\sqrt{2}}$$

37.$ x^{2}+\left(\sqrt{5}-\sqrt{2}\right)x+\sqrt{10}=0$

$$\left(x+\sqrt{5}\right)\left(x-\sqrt{2}\right)=0$$

$$\left(x+\sqrt{5}\right)=0$$

$$∴x=-\sqrt{5}$$

$$\left(x-\sqrt{2}\right)=0$$

$$∴x=\sqrt{2}$$