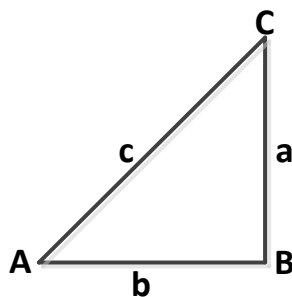


(09) 三角函數方程式

三角函數有很多方程式，同學們應該知道這些方程式都是很容易證明的。

(1) 試證 $\sin^2 \theta + \cos^2 \theta = 1$

見下圖：



$$\sin \theta = \frac{a}{c}$$

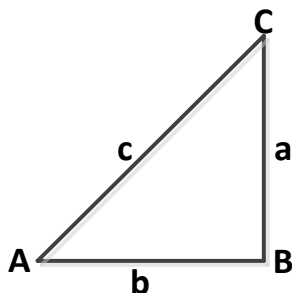
$$\cos \theta = \frac{b}{c}$$

故

$$\begin{aligned} & \sin^2 \theta + \cos^2 \theta \\ &= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} = 1 \end{aligned}$$

(2) 試證 $1 + \tan^2 \theta = \sec^2 \theta$

請看下圖：



$$\begin{aligned} & 1 + \tan^2 \theta \\ &= 1 + \frac{a^2}{b^2} = \frac{a^2 + b^2}{b^2} = \frac{c^2}{b^2} = \sec^2 \theta \end{aligned}$$

以下是同學們該記得的三角公式，而且要能導出這些公式：

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\tan(90^\circ - \theta) = \cot \theta$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

(3) 已知 $\sin \theta = \frac{1}{2}$ ，求 $\cos \theta$

〈方法 1〉

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\frac{1}{4} + \cos^2 \theta = 1$$

$$\cos^2 \theta = 1 - \frac{1}{4} = \frac{3}{4}, \quad \cos \theta = \frac{\sqrt{3}}{2}$$

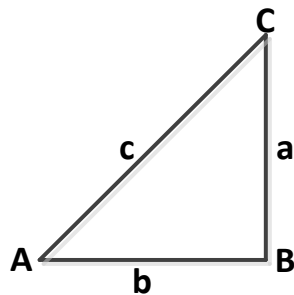
〈方法 2〉

$$\sin \theta = \frac{1}{2} = \frac{a}{c}$$

$$a = \frac{1}{2}c$$

$$b^2 = c^2 - a^2 = c^2 - \frac{1}{4}c^2 = \frac{3}{4}c^2$$

$$b = \frac{\sqrt{3}}{2}c, \quad \cos \theta = \frac{b}{c} = \frac{\sqrt{3}}{2}$$



(4) 已知 $\tan \theta = \frac{3}{4}$, 求 $\cos \theta$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\sec^2 \theta = 1 + \tan^2 \theta = 1 + \left(\frac{3}{4}\right)^2 = 1 + \frac{9}{16} = \frac{25}{16}$$

$$\sec \theta = \frac{5}{4}$$

$$\cos \theta = \frac{4}{5}$$